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Praastrup, U; Ibsen, Lars Bo; Lade, Poul V.

Publication date:
1999

Document Version
Early version, also known as pre-print

AAU Geotechnical Engineering Papers

ISSN 1398-6465 R 9906

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June 1999

Soil Mechanics Paper No 29

GEOTECHNICAL ENGINEERING GROUP
AALBORG UNIVERSITY DENMARK

Praastrup, U., Ibsen, L.B., Lade, P.V. (1999). Presentation of Stress Points in the Customised Octahedral Plane.

AAU Geotechnical Engineering Papers, ISSN 1398-6465 R9906.

Soil Mechanics Paper No 29

The paper has been published in *Proc.13th ASCE Engineering Mechanics Division Conference,* June 1999, Baltimore, USA, 6 pages.

Presentation of Stress Points in the Customised Octahedral Plane

U. Praastrup, L. B. Ibsen
Aalborg University, Aalborg, Denmark.
i5ulrik@civil.auc.dk, i5lbi@civil.auc.dk
and
P. V. Lade
Johns Hopkins University, Baltimore, USA.
poul.lade@jhu.edu

Abstract:
It can be a difficult task to comprehend and interpret three dimensional data, such as the principal stresses measured during true triaxial or cubical tests. The paper introduces a new and improved octahedral plane at which the reading and plotting of three dimensional data have been greatly simplified compared to existing planes. On this basis, the plane is entitled the CO-plane, which is an abbreviation for Customised Octahedral Plane. At first, the paper gives a formal definition of the new plane and compares it with the π-plane, which is one of the classical planes. Secondly, it explains how data are plotted in the CO-plane and how data already plotted are read using two distinct methods. Thirdly, it demonstrates how the trace of a generic stress surface together with some experimental results are plotted in the plane.

1 Introduction

Understanding the stress-strain behaviour of geomatrials is for most researchers very complex. Typically, constitutive models are designed to capture some of the characteristics associated with the stress-strain behaviour of these materials. In the process of understanding the stress-strain behaviour and in particular the design of constitutive relations, various data have to be collected and in addition presented in diagrams for interpretation purposes. The diagrams in which the data are presented should be non-confusing and straight-forward without the loss of important information affording easy interpretation of data. This is in particular true when it comes to presenting three dimensional data, which is the topic of this paper.

In mechanics, we are most often interested in stresses and strains and consequently in the plotting of these quantities. In the paper only stresses are considered, although the findings may apply to other three dimensional data such as strains. In geomechanics we have to consider both total and effective stresses, which are most frequently presented in the form of the Cauchy or true stress measure. Only effective stresses of the Cauchy type are considered here. The Customised Octahedral Plane, the CO-plane, is a plane in which the three principal effective stress components of a second-order effective Cauchy stress tensor can be plotted. A formal definition of the plane is given in section 2 together with a discussion on some properties of the plane. Procedures for plotting and reading stress points are described in section 3. Section 4 introduces a generic stress surface (Praastrup et al. 1999). The trace of the surface and test results obtained by Lade and Duncan (1973) are plotted in the CO-plane.

2 Definition of the Customised Octahedral Plane

The stresses considered herein are denoted by σ_i, where the index refers to the directions of a three dimensional right-handed Cartesian coordinate system. The index can be any of the values 1, 2 or 3. The chosen coordinate system is recognised as the principal stress space, where compression is considered positive (Fig. 1a). In geomechanics we are accustomed to use primes for effective stresses, but these are neglected herein.

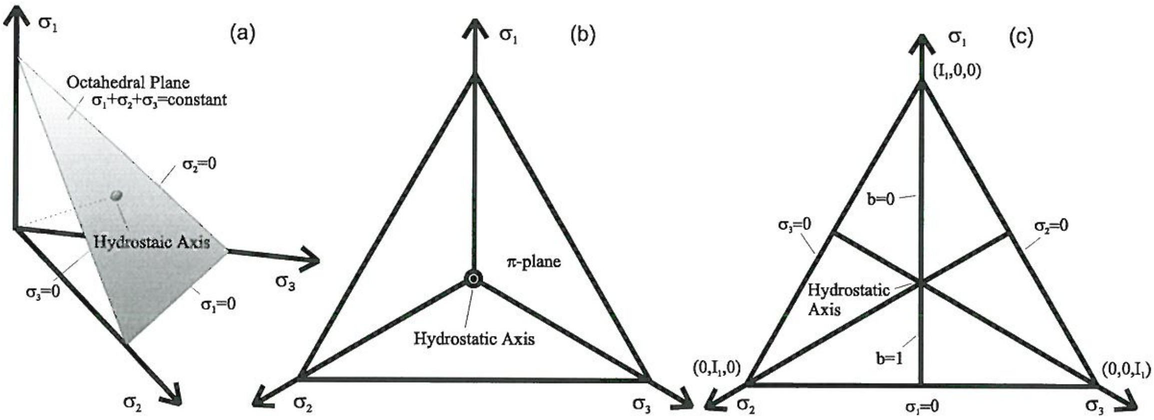

Figure 1. Octahedral planes. (a) The location of an octahedral plane in principal stress space. (b) The π-plane. (c) The Customised Octahedral plane.

A three dimensional stress point can, of course, be plotted in the principal stress space, but as one may have experienced, it is rather difficult to interpret data plotted herein. Instead three dimensional stress points are most often plotted in a certain plane of the principal stress space. In the literature (e.g. Chen and Saleed 1994) several planes or diagrams are suggested for plotting these data. The different planes all have their advantages and disadvantages and one has to chose the most appropriate diagram based on the problem at hand. Here, an important issue is whether the stress-strain behaviour of the material under investigation is independent or dependent on the mean effective stress. The mean effective stress, p, is defined as the average of the principal stresses. For all practical purposes it has no influence on the stress-strain behaviour of materials like steel. Stresses obtained from tests on such materials are, therefore, often plotted in the deviatoric plane (Chen and Saleed 1994). In geomechanics, the mean effective stress affects the stress-strain behaviour. Thus, the mean effective stress should be included in the chosen plane or diagram. Geomaterials such as sands and clays do not have effective cohesion unless they are truly cemented and cannot sustain zero or tensile principal stresses. Both tensile and compressive stresses are included in the principal stress space, but as we focus on compressive stresses, only the non-tensile octant of the principal stress space is considered. An octahedral plane is a plane where the sum of the principal stresses and hence the mean effective stress is constant as outlined in Figure 1a. The octahedral plane is written:

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 = \text{constant};\ \sigma_1 \geq 0;\ \sigma_2 \geq 0;\ \sigma_3 \geq 0 \tag{1}$$

Where I_1 is the first Cauchy invariant of the stress tensor or simply three times the mean effective stress. The octahedral plane in Figure 1a demonstrates that one of the principal stresses is zero at the boundary of the plane. Moreover, it shows that the octahedral plane reduces to an equilateral triangle due to the tension cut-off in (1). Octahedral planes come in different versions. Two examples, the π- and CO-plane, are shown in Figures 1b,c. The main difference between these two planes is the location of the principal axes spanning the principal stress space. In the π-plane, the axes are projected onto the plane as if one viewed the plane along the hydrostatic axis facing the origin (Fig. 1b). The hydrostatic axis is in principal stress space found as the trace of stress points having equal components. As seen in Figure 1b, the origin of the principal stress space is in the π-plane located on the hydrostatic axis. The location is misleading, as the zero stress condition is not present in an octahedral plane unless I_1 is zero. This corresponds to a single point in principal stress space, which can be seen from (1).

The CO-plane solves this problem by placing the axes such that a principal stress is indeed zero on the boundary of the plane as shown in Figure 1c. On each side of the triangle one of stress components is zero. Therefore, one zero point is placed at the midpoint of each side. At each vertex of the triangle one of the stress components is I_1, while the other components are zero as shown in Figures 1a,c. Consequently, an axis is placed so that it passes through the vertex of the triangle, where it attains the value of I_1. The three principal axes intersect at a point, which corresponds to intersection between the hydrostatic axis and the CO-plane. Here, all stress components attain the value of $I_1/3$. The location of the principal axes and the expression in (1) define the CO-plane.

2.1 *CO-plane Properties*

The intermediate principal stress plays an important role in the stress-strain behaviour of soils (Lade and Duncan 1973). Major, intermediate and minor principal stresses are denoted by σ_I, σ_{II} and σ_{III}, respectively. The relative magnitude of the intermediate stress is often indicated by the quantity b (Habib 1953):

$$b = (\sigma_{II} - \sigma_{III})/(\sigma_I - \sigma_{III}) \tag{2}$$

Triaxial tests are traditionally performed on cylindrical specimens, where two of the three stress components are equal. The expression in (2) shows that b is zero in triaxial compression tests and unity in triaxial extension tests. In the CO-plane, the stress paths of such tests coincide with the principal axes plotted in the plane as two stress components are equal on an axis. The point corresponding to a hydrostatic state of stress divides an axis into two parts having a length of $2I_1/3$ and $I_1/3$, respectively. The longer part corresponds to b=0, whereas the smaller part corresponds to b=1 (Fig. 1c). The lines of b=0 and b=1 divide the plane into six regions in which the directions for the major, intermediate and minor stress change.

3 Plotting and Reading Data

One of the major advantages of the CO-plane is the ease with which a stress point is plotted and read. This section explains how data are plotted and read in the plane using two different approaches; (1) reading and plotting by hand and (2) reading and plotting by pseudo-coordinates.

3.1 *Reading and Plotting by Hand*

The location of the principal axes simplifies the manner in which to read and plot a stress point. The reading of a stress point follows two rules, which are stated below.

- Rule no. 1: *If the stress point is located on axis σ_i, then the value of the corresponding component (σ_i) is read directly on the axis.*
- Rule no. 2: *The value of a component σ_i, of a stress point positioned off the corresponding axis is found as follows; (1) draw a line from the stress point perpendicular to the axis and then (2) read the value directly on this axis.*

The reading of the components of a stress point is exemplified in Figure 2a. In the figure a full circle indicates a stress point, whereas a solid square indicates the position for reading the value of a specific component. Applying rule no. 2 to the stress point in Figure 2a enables us to obtain the following principal stress components ($2I_1/3$, 0, $I_1/3$).

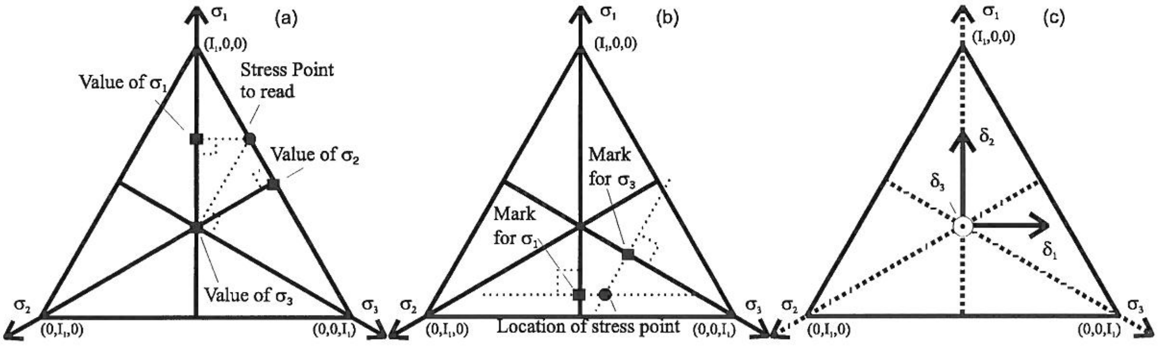

Figure 2. Examples on (a) Reading a stress point and (b) Plotting a stress point. (c) Pseudo-coordinates

The reading of a stress point is seemingly very simple. This is also true when plotting a stress point for which the value of I_1 corresponds to the CO-plane in question. Here, one has to follow one of the three rules listed below.

- Rule no. 1: *All components are equal: Plot the stress point where the axes intersect.*
- Rule no. 2: *Two components are exactly equal: Plot the stress point on the axis corresponding to third stress component.*
- Rule no. 3: *None of the components are equal: Select two components and mark their positions on their respective axes. Then draw a line for each mark, perpendicular to their axes. Plot the stress point where the two lines intersect.*

An example of how to plot a stress point is illustrated in Figure 2b. The stress point to be plotted is ($I_1/10$, $4I_1/10$, $5I_1/10$). Rule no. 3 applies as none of the components are equal. At first, it states that two components have to be chosen. In Figure 2b it is seen that σ_1 and σ_3 have been selected. Secondly, it states that a mark has be to set for each of the two components on their respective axes. Thirdly, two lines are drawn so that they are perpendicular to their respective axes. The stress point is finally plotted at the intersection between the two lines as outlined in Figure 2b.

3.2 *Pseudo-coordinates*

The above mentioned procedures are easily implemented when a few stress points are to be plotted or read, but when it comes to plotting and reading many stress points another procedure should be followed. This directs us to the use of pseudo-coordinates, which are denoted by δ_i. As shown in Figure 2c, the origin of the pseudo-coordinate system is positioned in the CO-plane where the hydrostatic axis intersects the plane. Using the standard rules for transforming a position vector (Spencer 1980), like the position vector for a stress point, it is straight-forward to obtain the transformation rule shown below.

$$\delta_1 = \frac{1}{\sqrt{3}}(\sigma_3 - \sigma_2);\ \delta_2 = \frac{1}{3}(2\sigma_1 - \sigma_2 - \sigma_3);\ \delta_3 = \frac{\sqrt{2}}{3}(\sigma_1 + \sigma_2 + \sigma_3) \tag{4}$$

As I_1 is constant in any CO-plane, only the first two components in (4) are needed in order to plot or read a given stress point. Therefore, the last expression in (4) can be ignored when the plotting or reading of a stress point is carried out using the pseudo-coordinates. However, the presence of the last expression in (4) emphasises that stress points with different mean effective stresses cannot be plotted in the same plane, unless a suitable projection scheme is employed.

4 Examples

This section presents two examples to place the above findings into a more practical context. The first example concerns a generic stress surface which can operate as yield surface, plastic potential surface or failure surface in an elasto-plastic constitutive model. The second example concerns the plotting of a series of failure points obtained by Lade & Duncan (1973).

The development of the generic stress surface has been inspired by the work of Krenk (1996) and Lade (1977). The generic stress surface is written (Praastrup et al. 1999):

$$(\sigma_1 - a)(\sigma_2 - a)(\sigma_3 - a) = \alpha(p - a)^3;\ \sigma_i \geq 0 \tag{5}$$

where a and α are material parameters to be determined experimentally. The stress surface is termed generic as it for all practical purposes has the ability to transform from a triangle to a circle in the CO-plane, which is shown in Figure 3a.

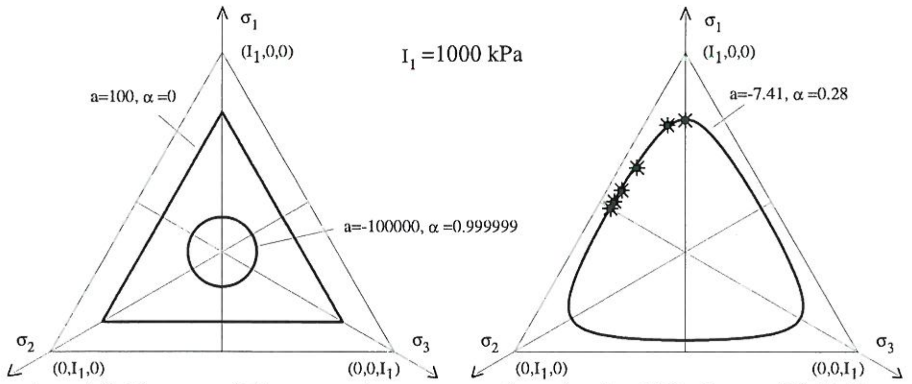

Figure 3. Examples. (a) Traces of the generic stress surface in the CO-plane. (b) Failure points for dense Monterey No. 0 Sand (Lade & Duncan 1973).

Moreover, the stress surface is termed generic as it can capture some of the stress surfaces presented in the literature. The failure criterion proposed by Lade (1977) is for example captured by choosing the following set of parameters:

$$a = 0;\ \alpha = \left(\frac{\eta}{27\zeta} + 1\right)^{-1} \ for\ \zeta = \left(\frac{I_1}{p_a}\right)^m \tag{6}$$

where p_a is the atmospheric pressure. η and m are non-negative constant material parameters, which are determined experimentally. Figure 3b shows a series of failure points obtained from true triaxial tests on dense specimens of Monterey No. 0 Sand. The specimens were sheared from the hydrostatic axis while keeping the cell pressure and b in (2) constant. Keeping these quantities constant during shear, causes the mean effective stress to vary during a test and moreover in-between tests.

As noted in connection with (4) the observed failure points and in particular the stress paths cannot be plotted in the same CO-plane unless a suitable projection scheme is applied. As the failure envelope for the sand is curved it is necessary to include this curvature in the projection scheme. Here, the failure points are projected keeping both the material parameter m in (6) and b in (2) constant. Subsequently, the generic stress surface in (5) is calibrated and plotted together with the failure points as shown in Figure 3b. The plotting was carried out using the first two pseudo-coordinates in (4). Figure 3b reveals that the generic stress surface captures the failure points very well.

5 Conclusion

The paper introduces an improved octahedral plane called the Customised Octahedral Plane. The plane is quite similar to other octahedral planes, but a different position of the principal axes enables the user to interpret three dimensional data in a more straightforward manner. The plotting and reading of data such as the principal stresses obtained from true triaxial tests is very simple. The ease with which three dimensional data is plotted and read is probably the greatest advantage of the plane. The most complicated task involved in the plotting or reading of a single stress point is the drawing of two straight lines. For plotting and reading of a series of stress points, the paper introduces a set of transformation rules, which may be employed in this case. Essentially, the set of transformation rules reduces a three dimensional stress point presentation to a two dimensional presentation as the mean effective stress and the first Cauchy invariant are inherently constant in a given CO-plane. This is evident from the pseudo-coordinates introduced in (4). The three pseudo-coordinates emphasises that in order to plot a series of stress points, initially with different mean effective stresses, it is necessary to project all the stress points on to a common CO-plane. The projection of such stress points is exemplified by calibrating a generic stress surface to a series of failure points initially using different mean effective stresses, which also highlights that the plane can be used in a more practical context.

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